

3.2 Higher Derivatives (minimum Homework: 1, 3, 5, 13, 15)

1) $f(x) = 12x^4 + 5x^3 + 3x^2 - 6x + 1$

a) $f'(x) = 4 \cdot 12x^3 + 3 \cdot 5x^2 + 2 \cdot 3x - 6$

$f'(x) = 48x^3 + 15x^2 + 6x - 6$

b) $f''(x) = 3 \cdot 48x^2 + 2 \cdot 15x + 6$

$f''(x) = 144x^2 + 30x + 6$

c) $f'''(x) = 2 \cdot 144x + 30$

$f'''(x) = 288x + 30$

$$3) f(x) = -3x^3 + 6x^2 + 8x + 9$$

$$a) f'(x) = 3(-3)x^2 + 2(6)x + 8$$

$$f'(x) = -9x^2 + 12x + 8$$

$$b) f''(x) = 2(-9)x + 12$$

$$f''(x) = -18x + 12$$

$$c) f'''(x) = -18$$

$$f'''(x) = -18$$

$$5) f(x) = 3x^{-5}$$

$$\begin{aligned} \text{a) } f'(x) &= -5(3)x^{-5-1} \\ &= -15x^{-6} \text{ or } -\frac{15}{x^6} \end{aligned}$$

$$f'(x) = -15x^{-6} = \frac{-15}{x^6}$$

$$\begin{aligned} \text{b) } f''(x) &= -6(-15)x^{-6-1} \\ &= \boxed{90x^{-7} = \frac{90}{x^7}} \end{aligned}$$

$$f''(x) = \frac{90}{x^7}$$

$$\begin{aligned} \text{c) } f'''(x) &= -7(90)x^{-7-1} \\ f'''(x) &= -630x^{-8} = \frac{-630}{x^8} \end{aligned}$$

$$f'''(x) = \frac{-630}{x^8}$$

$$7) f(x) = e^{2x+3}$$

$$a) f'(x) = \frac{d(2x+3)}{dx} e^{2x+3}$$

$$f'(x) = 2e^{2x+3}$$

$$f'(x) = 2e^{2x+3}$$

$$b) f''(x) = 2 \cdot \frac{d}{dx}(2x+3) e^{2x+3}$$

$$= 2(2) e^{2x+3}$$

$$= 4e^{2x+3}$$

$$f''(x) = 4e^{2x+3}$$

$$9) f(x) = e^{x^2}$$

$$\begin{aligned} \text{a) } f'(x) &= \frac{d}{dx}(x^2) \cdot e^{x^2} \\ &= 2x e^{x^2} \end{aligned}$$

$$f'(x) = 2xe^{x^2}$$

$$\text{b) } f''(x)$$

Product Rule

1st $2x$

2nd e^{x^2}

Deriv 2

Deriv $2xe^{x^2}$
(see ↑)

$$f''(x) = 2x \cdot 2xe^{x^2} + 2e^{x^2}$$

$$f''(x) = 4x^2 e^{x^2} + 2e^{x^2}$$

$$= e^{x^2} (4x^2 + 2)$$

$$= 2e^{x^2} (2x^2 + 1)$$

$$f''(x) = 2e^{x^2} (2x^2 + 1)$$

$$11) f(x) = \ln(2x)$$

$$a) f'(x) = \frac{\frac{d}{dx} 2x}{2x} = \frac{2}{2x} = \frac{1}{x}$$

$$f'(x) = \frac{1}{x}$$

$$b) f''(x)$$

$$\begin{aligned} f'(x) &= 1x^{-1} \\ f''(x) &= -1 \cdot x^{-1-1} \\ f'''(x) &= -1x^{-2} \\ f^{(4)}(x) &= \frac{-1}{x^2} \end{aligned}$$

$$f''(x) = -1/x^2$$

$$13) f(x) = \ln(x^2)$$

$$a) f'(x) = \frac{\frac{d}{dx}(x^2)}{x^2} = \frac{2x}{x^2} = \frac{2}{x}$$

$$f'(x) = \frac{2}{x}$$

$$b) f''(x)$$

$$f'(x) = 2x^{-1}$$

$$f''(x) = -1 \cdot 2x^{-1-1}$$

$$f''(x) = -2x^{-2}$$

$$f''(x) = \frac{-2}{x^2}$$

$$f''(x) = -2/x^2$$

$$15) f(x) = \frac{x+2}{x-3}$$

$$a) f'(x)$$

Denom	$x-3$	Num	$x+2$
Deriv	1	Deriv	1

$$f'(x) = \frac{(x-3)(1) - 1(x+2)}{(x-3)^2}$$

$$= \frac{x-3-x-2}{(x-3)^2}$$

$$f'(x) = \frac{-5}{(x-3)^2}$$

$$b) f''(x)$$

$$f'(x) = \frac{-5}{(x-3)^2}$$

$$f''(x)$$

Denom	$(x-3)^2$	Num	-5
deriv	$2 \cdot \frac{d}{dx}(x-3)(x-3)'$	Deriv	0
	$= 2 \cdot 1(x-3)$		
	$= 2(x-3)$		

$$f''(x) = \frac{(x-3)^2(0) - (-5)(2)(x-3)}{((x-3)^2)^2}$$

$$f''(x) = \frac{10}{(x-3)^3}$$

$$f''(x) = \frac{10 \cancel{(x-3)}}{(x-3)^{\cancel{4} 3}} = \frac{10}{(x-3)^3}$$

$$17) f(x) = \frac{2x}{x+8}$$

a) $f'(x)$

Denom $x+8$

num $2x$

Deriv 1

Deriv 2

$$f'(x) = \frac{2(x+8) - 1(2x)}{(x+8)^2}$$

$$f'(x) = \frac{2x+16-2x}{(x+8)^2}$$

$$f'(x) = \frac{16}{(x+8)^2}$$

$$f'(x) = \frac{16}{(x+8)^2}$$

b) $f''(x)$

f'' Denom $(x+8)^2$

num 16

Deriv $2 \cdot \frac{d}{dx} (x+8)(x+8)'$
 $= 2 \cdot 1(x+8)$
 $= 2(x+8)$

Deriv 0

$$f''(x) = \frac{-32}{(x+8)^3}$$

$$f''(x) = \frac{0(x+8)^2 - 16(2)(x+8)}{((x+8)^2)^2}$$

$$f''(x) = \frac{-32(x+8)}{(x+8)^4}$$

$$f''(x) = \frac{-32}{(x+8)^3}$$